2 - 8
Proving Angle Relationships

Protractor Postulate

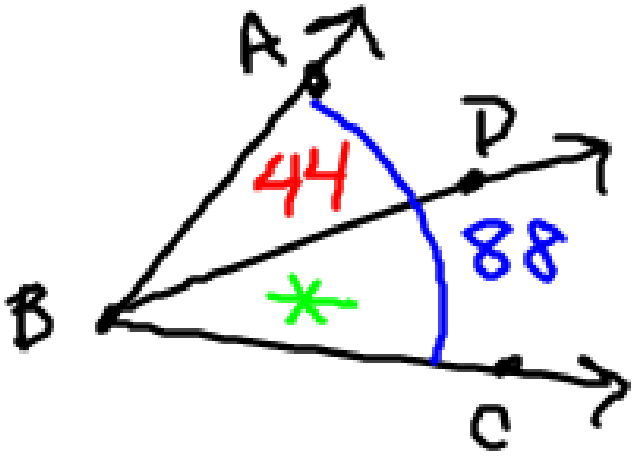
Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.

Angle Addition Postulate:

If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle QRS = m\angle PQS$.

If $m\angle PQR + m\angle QRS = m\angle PQS$, then R is in the interior of $\angle PQS$.

Ex: If $m\angle ABD = 44$ and $m\angle ABC = 88$, find $m\angle DBC$.



44°

Theorem 2.3: ***Supplement Theorem***

If two angles form a linear pair, then they are supplementary.

Theorem 2.4: ***Complement Theorem***

If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

Ex: If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 67$, find $m\angle 1$.

$m\angle 1 + m\angle 2 = 180$

$\_\_ + 67 = 180$

$113°$

Theorem 2.5:

Congruence of angles is reflexive, symmetric, and transitive.

Reflexive: $\angle 1 \cong \angle 1$

Symmetric: If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive: If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

Prove the symmetric property...

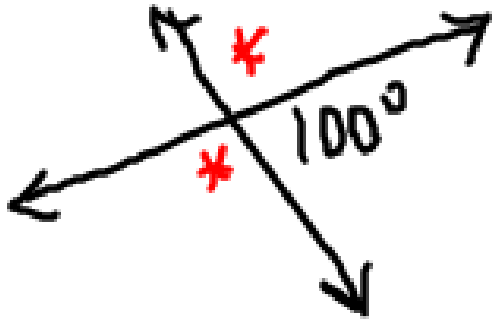
Given: $\angle A \cong \angle B$

Prove: $\angle B \cong \angle A$

Statements	Reasons
1. $\angle A \cong \angle B$	1. Given
2. $m\angle A = m\angle B$	2. defn. of $\cong$
3. $m\angle B = m\angle A$	3. symmetric
4. $\angle B \cong \angle A$	4. defn. of $\cong$

Theorem 2.6:

Angles supplementary to the same angle or congruent angles are congruent.



Theorem 2.7:

Angles complementary to the same angle or to congruent angles are congruent.

Given: $\angle 1$ and $\angle 3$ are complementary
$\angle 2$ and $\angle 3$ are complementary

Prove: $\angle 1 \cong \angle 2$

Statements	Reasons
1. $\angle 1, \angle 3$ comp $\angle 2, \angle 3$ comp	1. Given
2. $m\angle 1 + m\angle 3 = 90$ $m\angle 2 + m\angle 3 = 90$	2. defn. of comp.
3. $m\angle 1 + m\angle 3 = m\angle 2 +$ $-m\angle 3 \quad -m\angle 3$	3. substitution
4. $m\angle 1 = m\angle 2$	4. subtraction
5. $\angle 1 \cong \angle 2$	5. defn. of $\cong$

Given: $\angle 1$ and $\angle 2$ form a linear pair
$\angle 2$ and $\angle 3$ form a linear pair

Prove: $\angle 1 \cong \angle 3$

Statements	Reasons
1. $\angle 1, \angle 2$ linear $\angle 2, \angle 3$ linear	1. Given
2. $\angle 1, \angle 2$ supp. $\angle 2, \angle 3$ supp.	2. Thm. 2.3
3. $\angle 1 \cong \angle 3$	3. Thm. 2.6

Ex: If $\angle 1$ and $\angle 2$ are vertical angles and $m\angle 1 = x$ and $m\angle 2 = 228 - 3x$, find $m\angle 1$ and $m\angle 2$.

$m\angle 1 = m\angle 2$

$x = 228 - 3x$

$+3x \qquad +3x$

$\frac{4x}{4} = \frac{228}{4}$

$x = 57$

57°, 57°

Theorem 2.9:

Perpendicular lines intersect to form four right angles

Theorem 2.10:

All right angles are congruent.

Theorem 2.11:

Perpendicular lines form congruent adjacent angles.

Theorem 2.12:

If two angles are congruent and supplementary, then each angle is a right angle.

Theorem 2.13:

If two congruent angles form a linear pair, then they are right angles.

Homework:

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